

## Sec. 9.5 Completing the Square

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$x^2 + \overset{cx+cx}{2cx} + c^2 = (x+c)^2$$

$$\frac{2cx}{2x} = c \xrightarrow{c^2}$$

Problem 1: What is the value of  $c$  such that the following is a perfect square?

$$a. \quad x^2 - \overset{-10x-10x}{20x} + c = \boxed{(x-10)^2}$$

$$c = 100$$

$$(x-10)(x-10)$$

$$x^2 - 10x - 10x + \underline{100}$$

$$b. \quad x^2 - 8x + c = (x-4)^2$$

$$c = 16$$

$$c. \quad x^2 + 10x + \boxed{c} = (x+5)^2$$

$$c = 25$$

$$x^2 + bx + c = \left(x + \frac{b}{2}\right)^2$$

$$c = \left(\frac{b}{2}\right)^2$$

Problem 2: What are the solutions of the equation?

$$x^2 = 9$$

$$x = \pm 3$$

a.  $x^2 - 12x - 20 = 0$

$$\frac{x^2 - 12x + 36 = 20 + 36}{+20 \quad +20}$$

$$(x - 6)^2 = 56$$

$$\sqrt{(x - 6)^2} = \pm \sqrt{56}$$

$$x - 6 = \pm \sqrt{56}$$

$$x = 6 \pm \sqrt{56}$$

$$x = 6 + \sqrt{56} = 13.48$$

$$x = 6 - \sqrt{56} = -1.48$$

b.  $x^2 - 2x - 35 = 0$

$$\frac{x^2 - 2x + 1 = 35 + 1}{+35 \quad +35}$$

$$(x - 1)^2 = 36$$

$$\sqrt{(x - 1)^2} = \pm \sqrt{36}$$

$$x - 1 = \pm 6$$

$$x = 1 \pm 6$$

$$x = 1 + 6 = 7$$

$$x = 1 - 6 = -5$$

c.  $x^2 - 3x - 40 = 0$

$$\frac{x^2 - 3x + \frac{9}{4} = \frac{160}{4} + \frac{9}{4}}{+40 \quad +40}$$

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \pm \sqrt{\frac{169}{4}}$$

$$x - \frac{3}{2} = \pm \frac{13}{2}$$

$$x = \frac{3}{2} \pm \frac{13}{2}$$

$$x = \frac{3+13}{2} = \frac{16}{2} = 8$$

$$x = \frac{3-13}{2} = \frac{-10}{2} = -5$$

d.  $\frac{2y^2 - 8y - 10}{2} = \frac{0}{2}$

$$y^2 - 4y - 5 = 0$$

$$\frac{y^2 - 4y + 4 = 5 + 4}{+5 \quad +5}$$

$$(y - 2)^2 = 9 \rightarrow \pm \sqrt{\quad}$$

$$\sqrt{(y - 2)^2} = \pm \sqrt{9} \quad \pm \text{const.}$$

$$y - 2 = \pm 3$$

$$\frac{y - 2 = \pm 3}{+2 \quad +2}$$

$$y = 2 \pm 3 \begin{cases} 2 + 3 = 5 \\ 2 - 3 = -1 \end{cases}$$

$$e. 3x^2 + 18x = 21$$

Problem 3: Find the vertex by completing the square.  $y = |x+7| - 1$   
 $v: (-7, -1)$

$$a. \begin{array}{r} y = x^2 - 4x - 1 \\ +1 \end{array}$$

$$\frac{y+1+4}{y+5} = \frac{x^2 - 4x + 4}{(x-2)^2}$$

$$y - 5 = (x-2)^2$$

$$b. \begin{array}{r} y = x^2 + 6x - 7 \\ +7 \end{array}$$

$$\frac{y+7+9}{y+16} = \frac{x^2 + 6x + 9}{(x+3)^2}$$

$$y - 16 = (x+3)^2$$

$$y = (x+3) - 16$$

$$v: (-3, -16)$$

$$(x-2)(x-2)$$

$$x^2 - 2x - 2x + 4$$

$$y = (x-h)^2 + k$$

$$y = (x-2)^2 - 5$$

$$v: (h, k)$$

$$v: (2, -5)$$