

13.1 Exploring Periodic Data

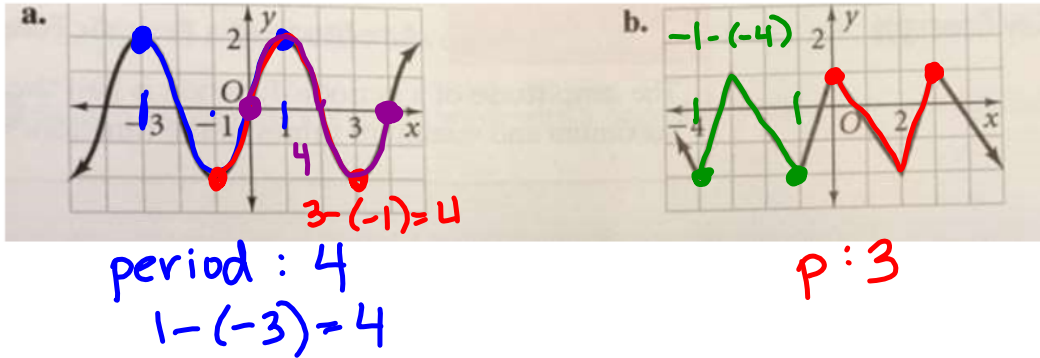
Periodic function - repeats a pattern of y-values (outputs) at regular intervals.

Cycle - 1 complete pattern. A cycle may begin at any point on the graph of the function.

Period - the horizontal length of 1 cycle, - in terms of x-values.

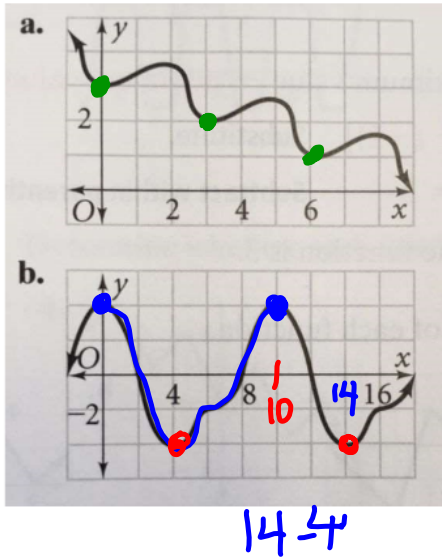
Example 1: Identifying Cycles and Periods

Analyze the periodic function. a) Identify 1 cycle in two different ways. b) Determine the period of the function.



Example 2: Identifying Periodic Functions

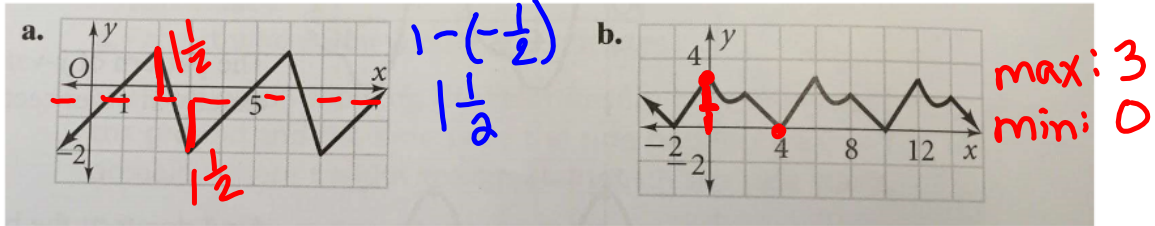
Determine whether each function is or is not periodic. If it is, find the period.



* $\frac{\text{max} - \text{min}}{2} = \text{Amplitude}$

Amplitude - of a periodic function measures the amount of variation in the y-values. To find the amplitude:
 ① Find average of the max + min = MIDLINE
 $\frac{\text{max} + \text{min}}{2} = y$ ② max - midline = amplitude
 min - midline = amplitude

Example 3: Find the Amplitude of the periodic function.



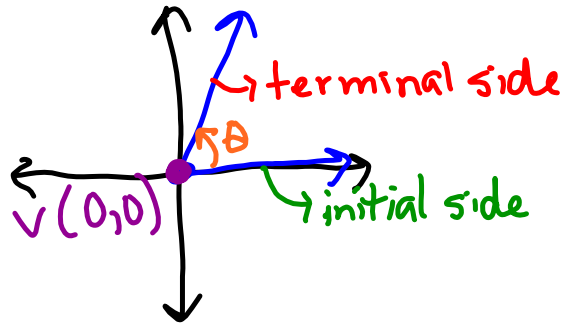
$\frac{1 + -2}{2} = -\frac{1}{2} = y$

$\frac{3 - 0}{2} = \frac{3}{2} = a$

13.2 Angles (day 1)

An angle in standard position has:

- vertex is at the (0,0)
- one ray is on the x-axis



Initial side - the ray on the positive x-axis

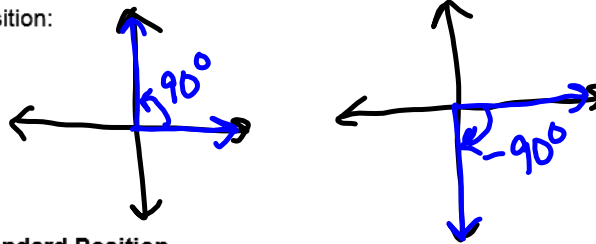
Terminal side - the other ray of the angle

The **measure of an angle in standard position** is the amount of rotation from the initial side to the terminal side.

The measure of an angle is **positive** when the rotation from the initial side to terminal side is counterclockwise

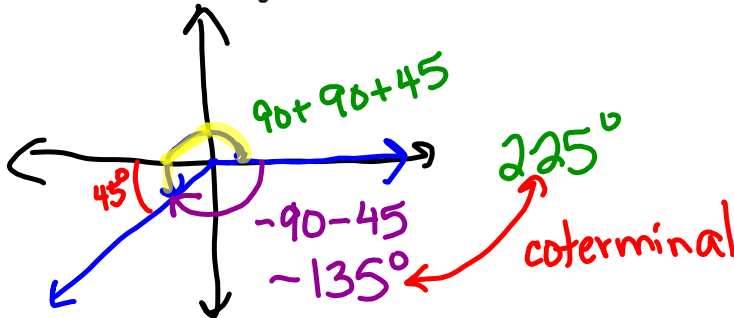
The measure of an angle is **negative** when the rotation from the initial side to terminal side is clockwise

Sketch an angle in standard position:



Example 1: Measuring an Angle in Standard Position

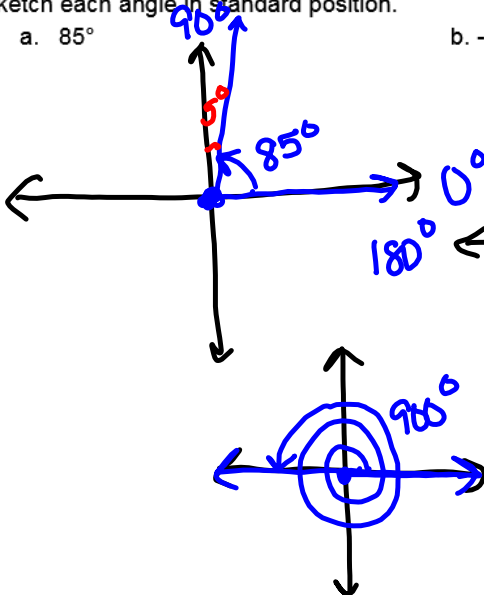
Find the measure of the angle.



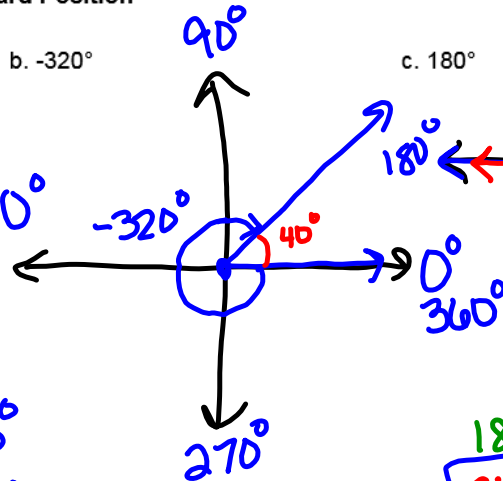
Example 2: Sketching an Angle in Standard Position

Sketch each angle in standard position.

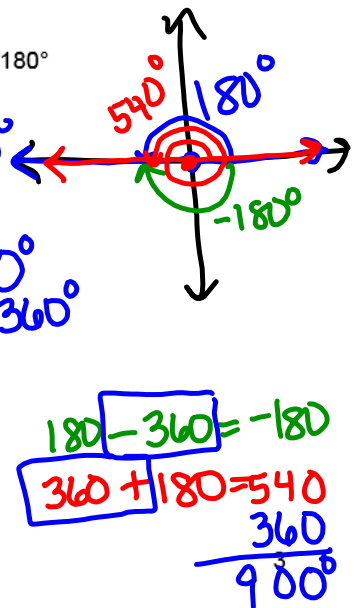
a. 85°



b. -320°



c. 180°



Coterminal Angles - Two angles in standard position are coterminal if they have the same terminal side. For any given angle, there are infinitely many coterminal angles. To find a coterminal angle, add or subtract 360° .

Example 3: Finding Coterminal Angles

a. Find a positive angle and a negative angle that are coterminal with 198° .

$$198 + 360 = 558^\circ$$

$$198 - 360 = -162^\circ$$

b. Are the angles with measure 40° and 680° coterminal? Explain.

$(680) - 360 = 320 \neq 360$
 $= 40^\circ$
 not coterminal
 $680 - 40 = 640 \div 360 = \times$

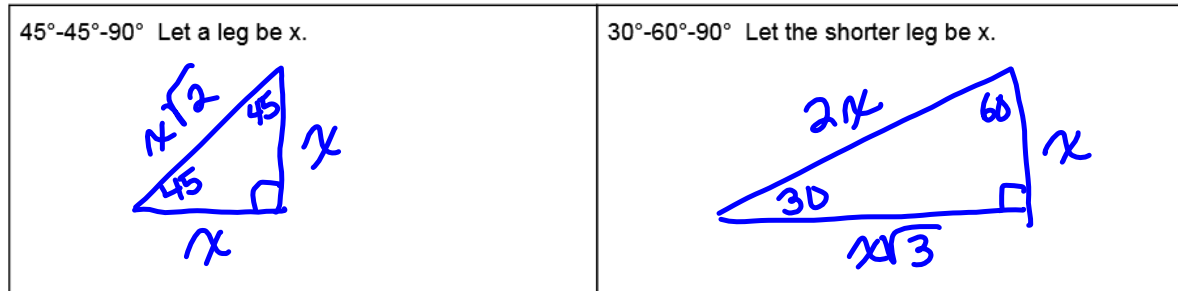
c. Find the measure of an angle between 0° and 360° coterminal with 385° .

$$385^\circ - 360^\circ = 25^\circ$$

d. Find the measure of an angle between 0° and 360° coterminal with -356° .


$$-356^\circ + 360^\circ = 4^\circ$$

Recall: Special Right Triangles



Find the missing side lengths in each 45°-45°-90° triangle. Rationalize any denominators.

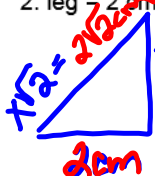
1. hypotenuse = 1 inch



$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\frac{\sqrt{2}}{2} = x$$

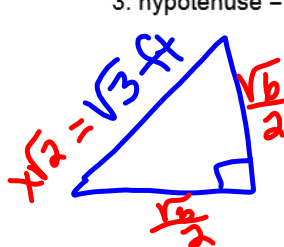
2. leg = 2 cm



$$\frac{x}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

$$x = 2$$

3. hypotenuse = $\sqrt{3}$ ft



$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$$

$$x = \frac{\sqrt{6}}{2}$$

$$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$$

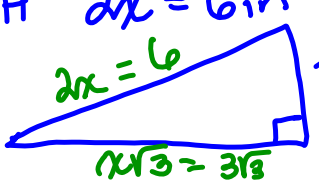
$$x = \frac{\sqrt{3} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$$

$$x = \frac{\sqrt{6}}{2}$$

Find the missing side lengths in each 30°-60°-90° triangle. Rationalize any denominators.

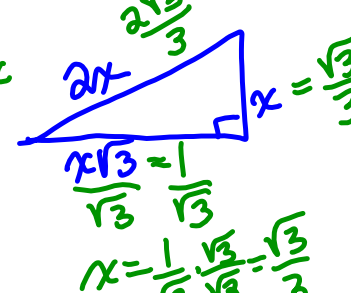
4. shorter leg = 3 inch

SL $x = 3$
 LL $x\sqrt{3} = 3\sqrt{3}$ in
 H $2x = 6$ in



5. longer leg = 1 cm

$$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$


6. hypotenuse = 1 ft

$$\frac{2x}{2} = \frac{1}{2}$$

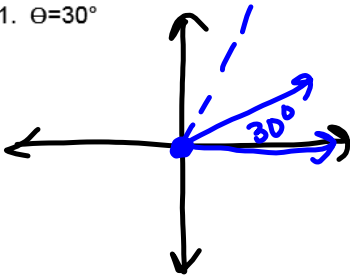
$$x = \frac{1}{2}$$

LL $x\sqrt{3} = \frac{1}{2} \cdot \sqrt{3} = \frac{\sqrt{3}}{2}$

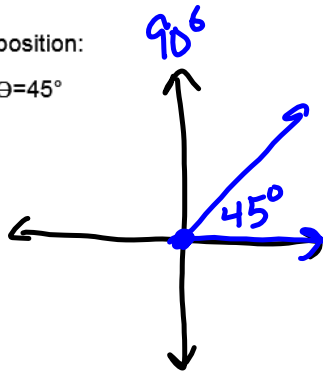
13.2 The Unit Circle (day 2)

Warm Up: Sketch the angle in standard position:

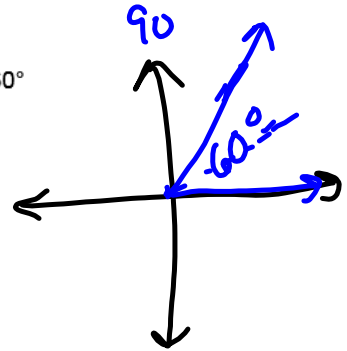
1. $\theta = 30^\circ$



2. $\theta = 45^\circ$



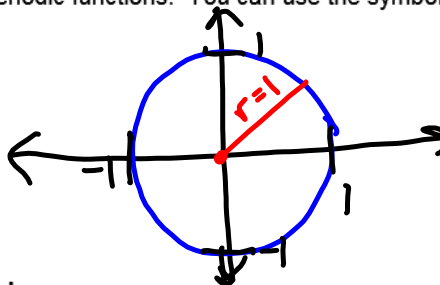
3. $\theta = 60^\circ$



Unit Circle - a circle with a radius of 1 and its center is at the vertex

Points on the unit circle are related to periodic functions. You can use the symbol θ "theta" for the measure of an angle in standard position.

Sketch the unit circle:



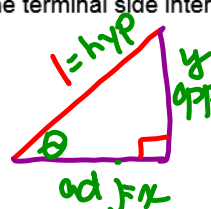
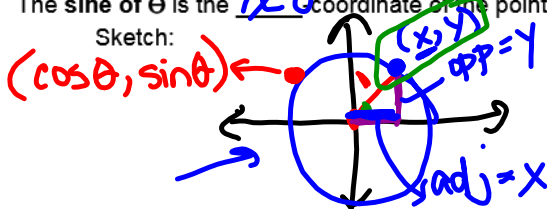
Definition: Cosine and Sine of an Angle

Given: an angle in standard position with measure θ .

The cosine of θ is the x-coordinate of the point at which the terminal side intersects the unit circle.

The sine of θ is the y-coordinate of the point at which the terminal side intersects the unit circle.

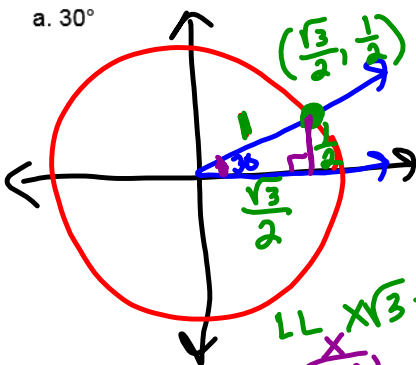
Sketch:



SOHCAHTOA
 $\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{1} = y$
 $\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{1} = x$

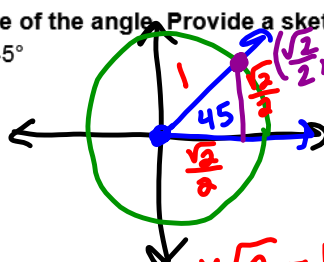
Example 4&5: Find the cosine and sine of the angle. Provide a sketch of the angle in standard position.

a. 30°

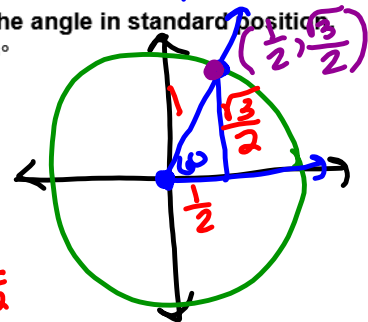


$\cos 30^\circ = \frac{\sqrt{3}}{2} = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$
 $\sin 30^\circ = \frac{1}{2} = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2} = \frac{1}{2}$

b. 45°



$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$
 $x = \frac{\sqrt{2}}{2}$
 $\cos 45^\circ = \frac{\sqrt{2}}{2}$
 $\sin 45^\circ = \frac{\sqrt{2}}{2}$



$\cos 60^\circ = \frac{1}{2}$
 $\sin 60^\circ = \frac{\sqrt{3}}{2}$

Find the cosine and sine of the angle. Provide a sketch of the angle in standard position.

d. 0°

e. 90°

f. 120°

g. -120°

h. -60°

i. 150°

Practice Problems:

Calculator Needed: For angles that are not a multiple of 30° or 45° , you will need your calculator. Find $\cos\theta$ and $\sin\theta$.

1. $\theta=32^\circ$

2. $\theta=-210^\circ$

3. $\theta=-10^\circ$

Find a positive and negative coterminal angle for the given angle.

4. $\theta=400^\circ$

5. $\theta=-125^\circ$

6. $\theta=-57^\circ$

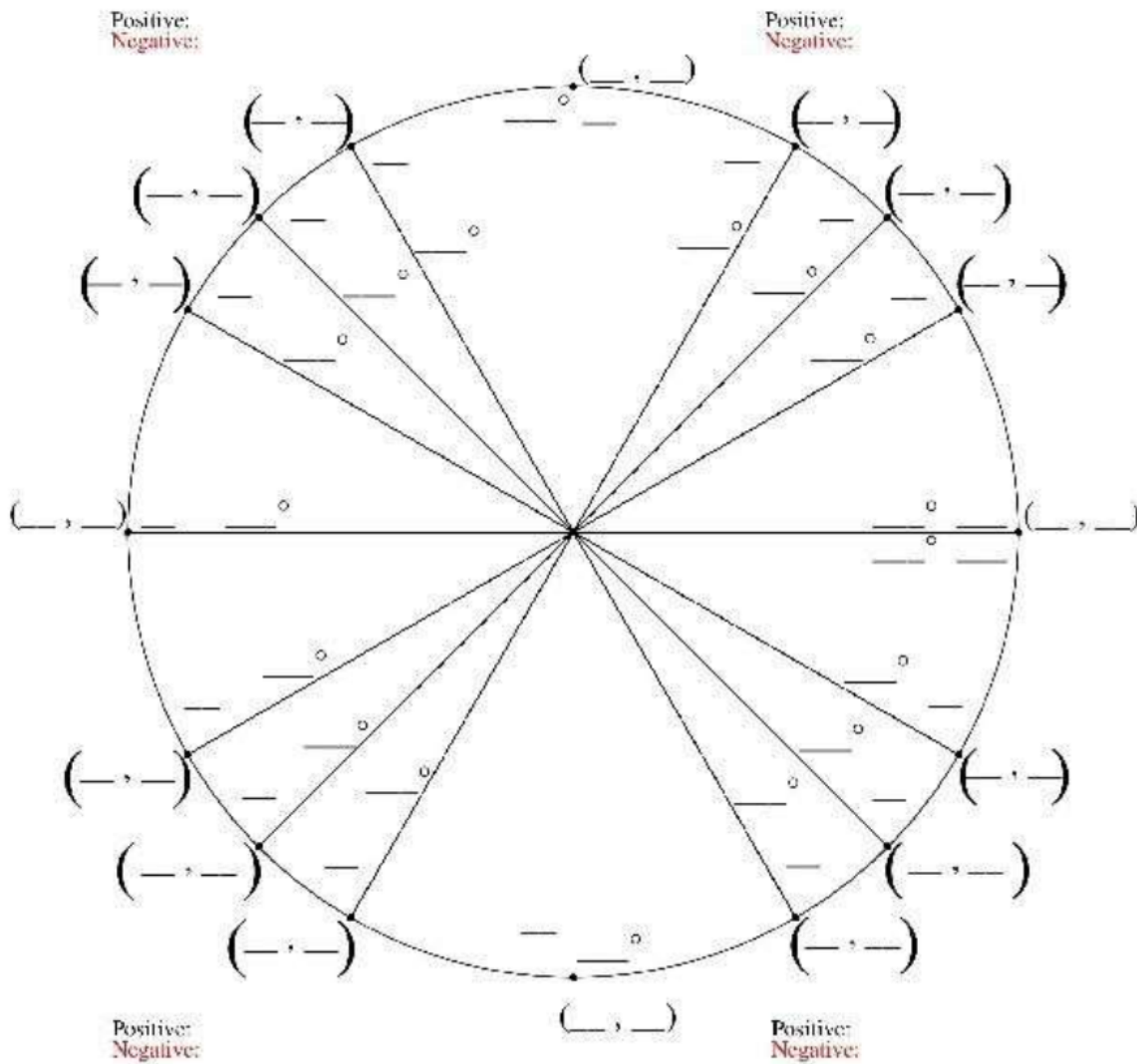
In which quadrant, or on which axis, does the terminal side of each angle lie? Sketch the angle to help you.

7. 210°

8. -60°

9. 270°

Fill in The Unit Circle



13.3 Radian Measure (Day 1)

Warm Up: Find the circumference of a circle with the given radius or diameter. Round your answer to the nearest tenth.

1. radius 4 in

2. diameter 70 m

Central angle - an angle whose vertex is the _____ of a circle *sketch:*

Intercepted arc - the portion of the circle with endpoints on the sides of the central angle and remaining points within the interior of the angle.

Radian - when the intercepted arc equals the radius, *sketch:* the measure of the angle is 1 radian

- The circumference of a circle is _____. Thus there are _____ in any circle.
- Since 2π radians = _____°, then π radians = _____°
- Thus you can use this proportion to convert between degrees and radians.

Example 1: Use a proportiona. Find the radian measure of 60° .b. Find the degree measure of $\frac{5\pi}{2}$ radians

Converting between Radians and Degrees

- To convert degrees to radians, multiply by _____
- To convert radians to degrees, multiply by _____

Example 2: Using Dimensional Analysis

Convert the angle to degrees. Round to the nearest degree.

a. $-\frac{3\pi}{4}$ radians

b. $\frac{\pi}{2}$ radians

c. 2 radians

Convert the angle to radians. Round to the nearest hundredth.

d. 27°

e. 225°

f. 150°

Example 3: Find the exact values of $\cos\theta$ and $\sin\theta$ for each angle measure.

Step 1: Convert to degrees.

Step 2: Draw the angle. The terminal side is the hypotenuse.

Step 3: Complete the right triangle. Draw a leg to the **x-axis**.

Step 4: State the $\cos\theta$ and $\sin\theta$.

a. $\frac{\pi}{4}$ radians

b. $\frac{\pi}{6}$ radians

c. $\frac{\pi}{2}$ radians

d. $\frac{5\pi}{6}$ radians

13.3 Arc Length (Day 2)

Warm Up: Convert the angle to degrees. Draw the angle in standard position, as well as the corresponding right triangle. Then find the exact values for $\cos\theta$ and $\sin\theta$.

$$\theta = \frac{\pi}{3}$$

You can find the length of an intercepted arc by using the proportion:

Length of an Intercepted Arc

For a circle of radius r and a central angle of measure θ (in *radians*), the length s of the intercepted arc is:

Example 4: Finding the Length of an Arc

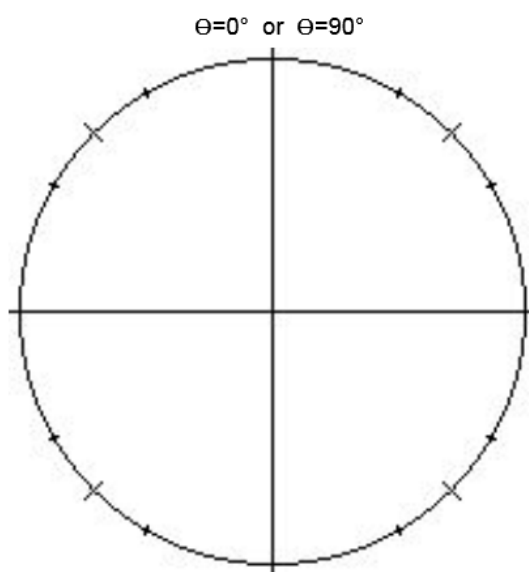
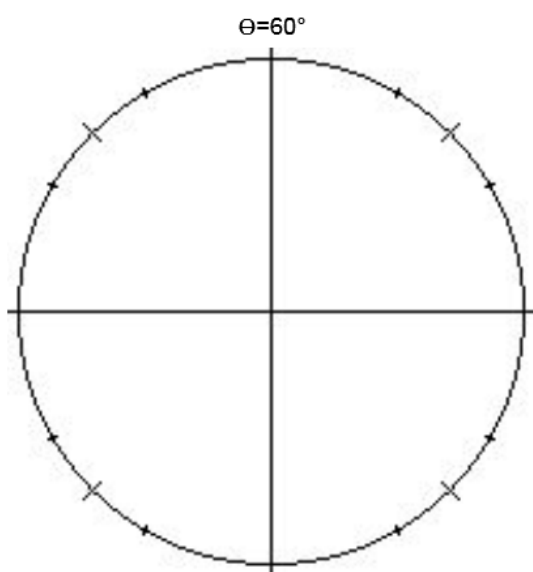
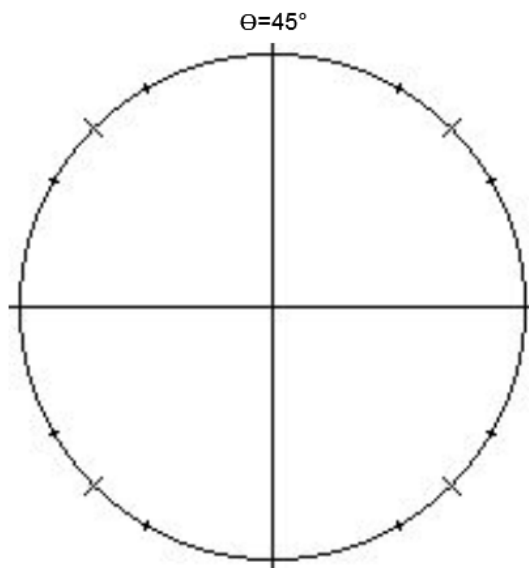
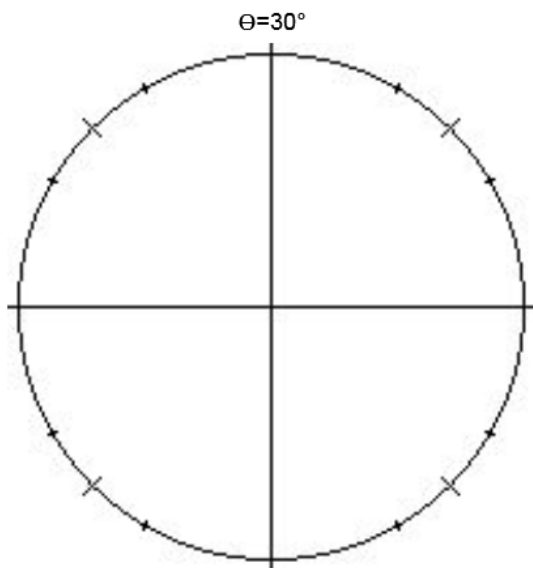
Find the length of the intercepted arc to the nearest tenth. Sketch a diagram!

a. Given: A circle of radius 3 in, $\theta = \frac{5\pi}{6}$.

b. Given: A circle of radius 5m, $\theta = \frac{2\pi}{3}$.

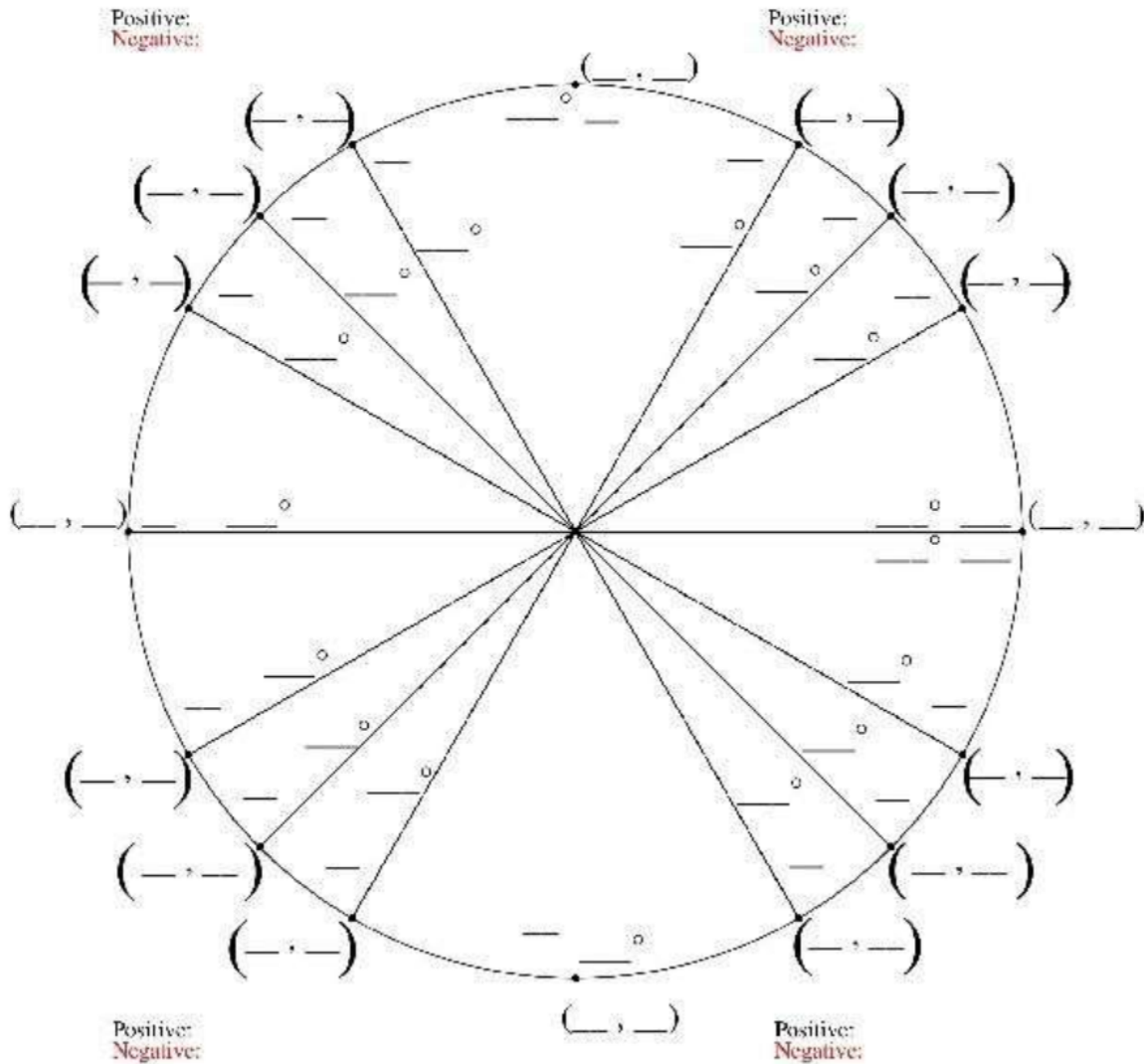
The Unit Circle: radius = _____

If you know quadrant 1, you can derive quadrants 2, 3, 4 by symmetry. Thus, let's study quadrant 1.



Now let's do all four quadrants...

Fill in The Unit Circle



Coordinates (x, y) on the unit circle:

$\cos\theta = \underline{\hspace{2cm}}$

$\sin\theta = \underline{\hspace{2cm}}$

$\tan\theta = \underline{\hspace{2cm}}$

13.4 The Sine Function (Day 1)

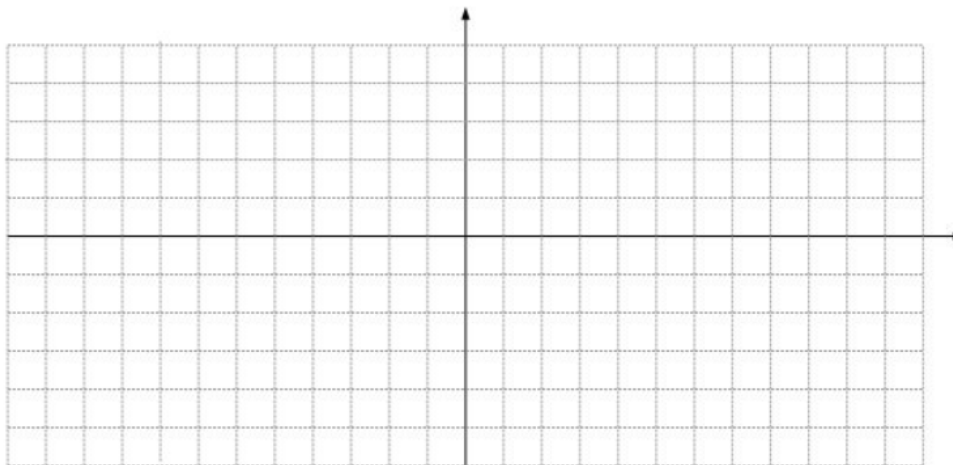
Warm Up: Use the graph. State:

1. the period
2. the domain
3. the amplitude
4. the range

sine function $y=\sin\theta$: for each measure of θ , the sine of θ corresponds with the _____-coordinates on the unit circle.

$y=\sin\theta$

θ (degrees)	0°	30°	45°	60°	90°	180°	270°	360°
y								



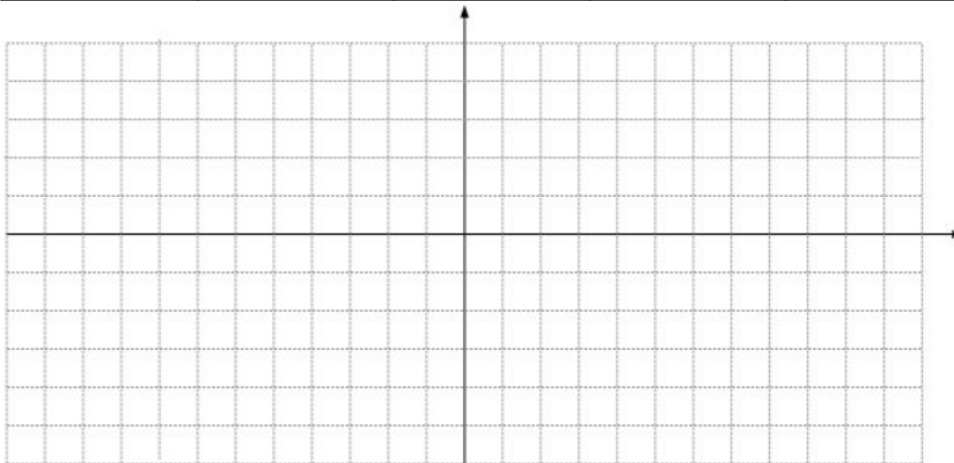
Example 1: Interpreting the Sine Function in Degrees

- a. What is the value of $y=\sin\theta$ for $\theta=270^\circ$?
- b. For what values of θ between 0° and 360° does the graph of $y=\sin\theta$ reach
 - o the maximum value of $y=1$?
 - o the minimum value of $y=-1$?
 - o x-intercept of $y=0$? aka "zero"

Mathematical convention: An angle measure θ can be expressed in degrees or in radians. However, when no unit is mentioned, you should use radians. Let's graph the sine function in radians...

$y = \sin \theta$

θ (radians)					
y					



Example 2: Estimating Sine Values in Radians

Use your graph above to estimate the value. Check your estimate with a calculator.

- a. $\sin 2$
- b. $\sin \pi$

For the sine function, find the following:

- a. amplitude
- b. period (in degrees and radians)
- c. domain and range

Properties of Sine Functions

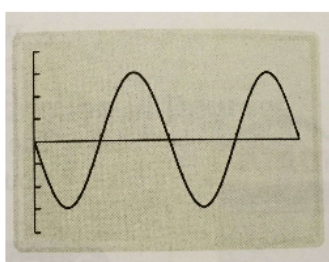
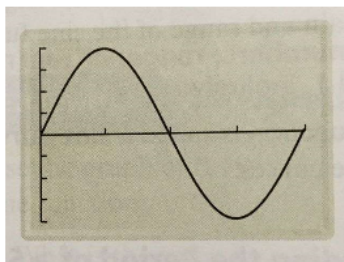
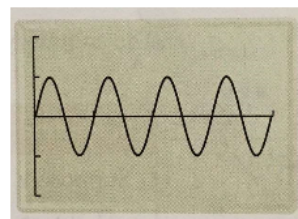
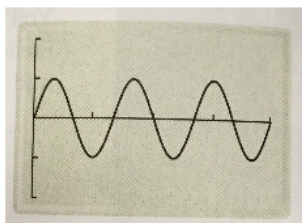
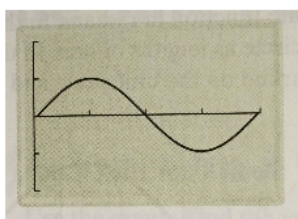
Suppose $y = a \sin b\theta$, where $a \neq 0$, $b > 0$, and θ in radians.

- The amplitude of the function is _____
- The number of cycles in the interval from 0 to 2π is _____
- The period of the function is _____

Examples 3&4: Finding the Period and Amplitude of a Sine Function

- a. Find the amplitude.
- b. How many cycles does the sine function have in the interval from 0 to 2π ?
- c. Find the period.

The θ -axis represents values from 0 to 2π . Each interval on the y-axis represents 1 unit.



13.4 The Sine Function (Day 2)

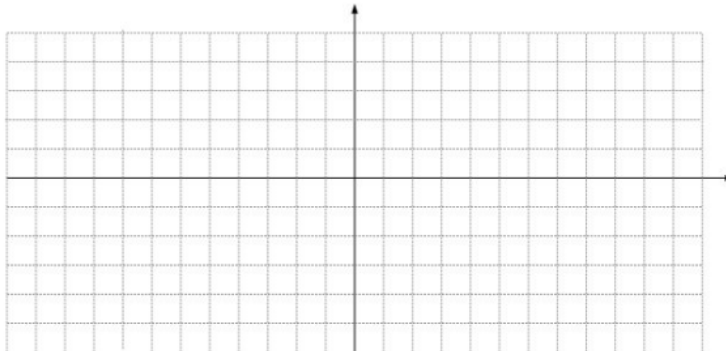
II. Graphing Sine Functions

You can use 5 points equally spaced through one cycle to sketch a sine curve. For $a > 0$, this 5-point pattern is zero-max-zero-min-zero.

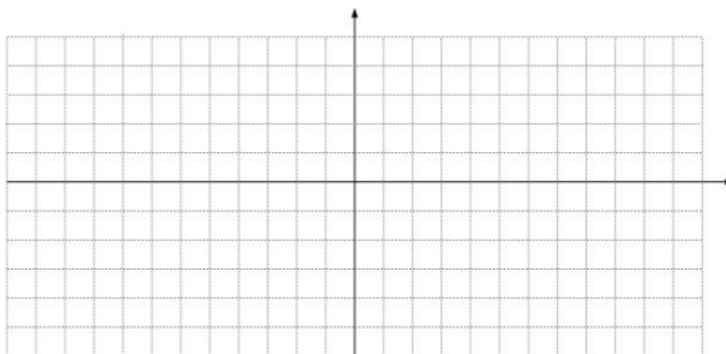
Example 5: Sketching a Graph

Sketch one cycle of each sine curve. Then write an equation for each graph.

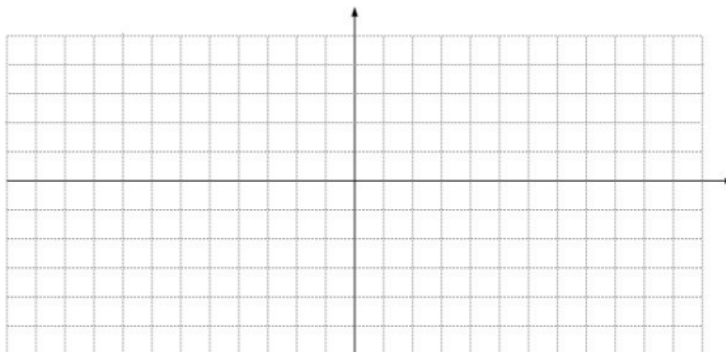
a. amplitude 2, period 4π , $a > 0$



b. amplitude 3, period 4, $a > 0$



c. Predict the 5-point pattern for the sine function when $a < 0$. Then sketch amplitude 2, period $2\pi/3$.



Example 6: Graphing from a Function Rule

Sketch one cycle of the following sine functions.

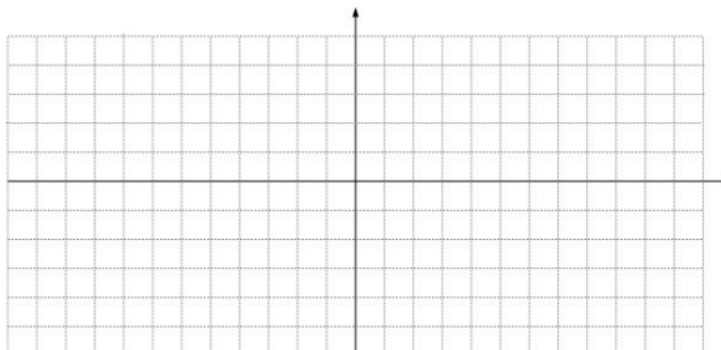
1. $y = \frac{1}{2}\sin 2\theta$

amplitude:

period:

interval spacing
on θ -axis:

5-point pattern:



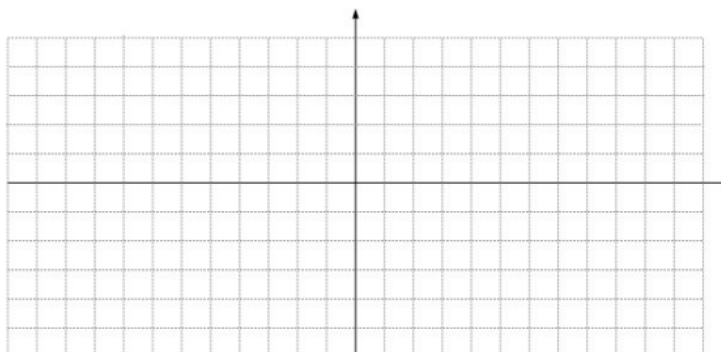
2. $y = 3\sin \frac{\pi}{2}\theta$

amplitude:

period:

interval spacing
on θ -axis:

5-point pattern:



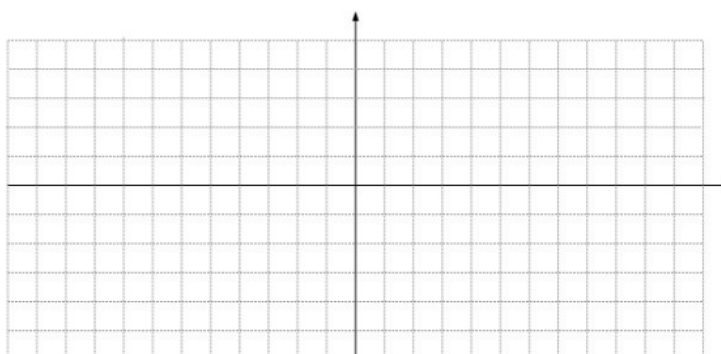
3. $y = -4\sin \frac{1}{2}\theta$

amplitude:

period:

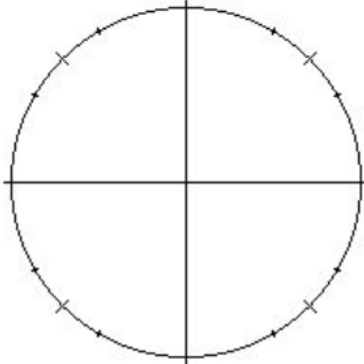
interval spacing
on θ -axis:

5-point pattern:



13.5 The Cosine Function (Day 1)

Warm up: Fill out the unit circle. Evaluate the following angles of cosine.

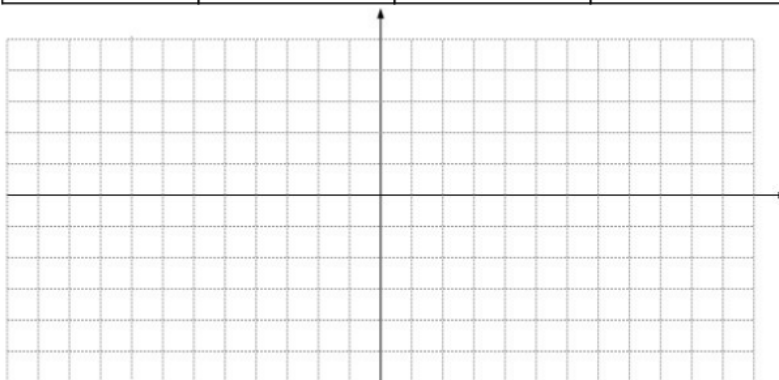


- 1. $\cos 0^\circ$ 2. $\cos 90^\circ$ 3. $\cos 180^\circ$
- 4. $\cos 270^\circ$ 5. $\cos 360^\circ$
- 6. $\cos 0$ 7. $\cos \frac{\pi}{2}$ 8. $\cos \pi$
- 9. $\cos \frac{3\pi}{2}$ 10. $\cos 2\pi$

Remember: $\cos\theta$ equals the _____-coordinate on the unit circle.

I. Graph the cosine function: $y=\cos\theta$

θ (radians)					
y					



Example 1: Interpreting the Graph of $y=\cos\theta$

a. Use your graph above. Find the following:

domain period range amplitude

b. In the interval from 0 to 2π , where do the maximum occur? minimum? zeros?

c. What is the 5-point pattern of the cosine graph?

Properties of Cosine Functions

Suppose $y = a \cos b\theta$, where $a \neq 0$, $b > 0$, and θ in radians.

- The amplitude of the function is _____
- The number of cycles in the interval from 0 to 2π is _____
- The period of the function is _____

Example 2: Sketching the Graph of a Cosine Function

Sketch one cycle of the following cosine functions.

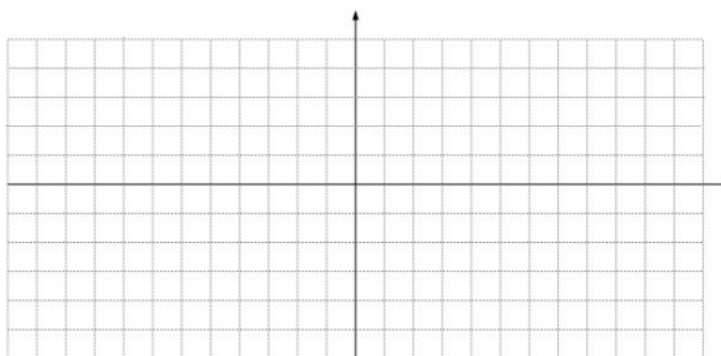
1. $y = \cos \frac{\pi}{2} \theta$

amplitude:

period:

interval spacing
on θ -axis:

5-point pattern:



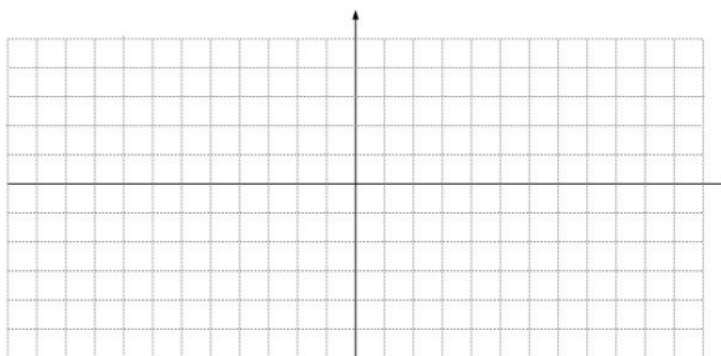
2. $y = -3 \cos \theta$

amplitude:

period:

interval spacing
on θ -axis:

5-point pattern:



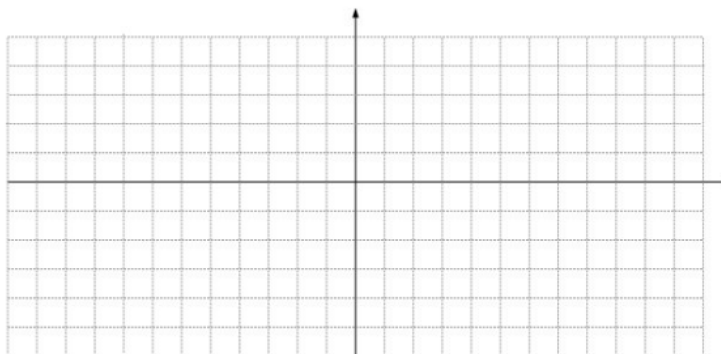
3. $y = 1.5\cos 2\theta$

amplitude:

period:

interval spacing
on θ -axis:

5-point pattern:



13.5 The Cosine Function (Day 2)

Warm up: Graph the following cosine functions.

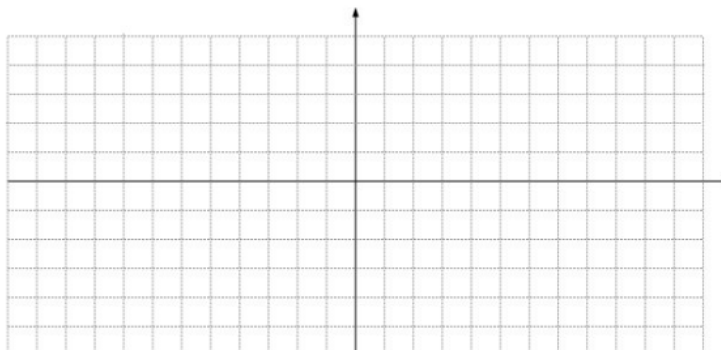
1. $y = 3\cos 2\theta$

amplitude:

period:

interval spacing
on θ -axis:

5-point pattern:



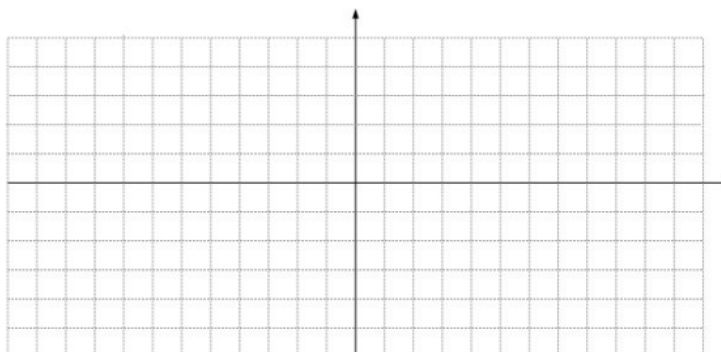
2. $y = -2\cos \theta$

amplitude:

period:

interval spacing
on θ -axis:

5-point pattern:



II. Solving Trigonometric Equations

Example 4: Solving a Cosine Equation

Solve the cosine equation in the interval from 0 to 2π . Round to the nearest hundredth. Calculators needed.

a. $-2\cos \theta = 1.2$

b. $3\cos 2t = -2$

c. $5\cos \frac{\pi}{2}t = 3$

Identify the period, range, and amplitude of each function.

22. $y = 3\cos\theta$

24. $y = 2\cos\frac{1}{2}t$

26. $y = 3\cos\left(-\frac{\theta}{3}\right)$

28. $y = 16\cos\frac{3\pi}{2}t$

13.6 The Tangent Function

Warm up: Use a calculator to find the sine and cosine of each θ . Then calculate the ratio of $\sin\theta$ to $\cos\theta$.

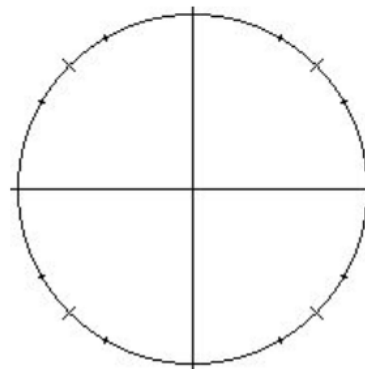
θ	$\sin\theta$	$\cos\theta$	$\frac{\sin\theta}{\cos\theta}$
1. $\frac{\pi}{3}$			
2. 30°			
3. 90°			
4. π			
5. $\frac{7\pi}{6}$			

I. The Tangent Function

The $\cos\theta$ is derived from the _____ - coordinate of the point on the unit circle.

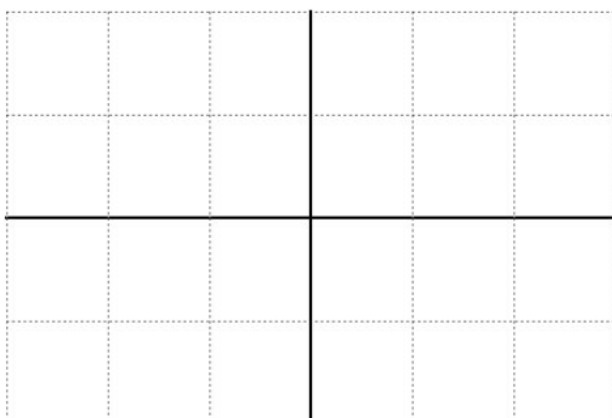
The $\sin\theta$ is derived from the _____ - coordinate of the point on the unit circle.

The $\tan\theta$ is derived from the ratio of $\sin\theta$ to $\cos\theta$. In other words: $\tan\theta = \underline{\hspace{2cm}}$



$y = \tan\theta$

θ (radians)	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$-\frac{\pi}{4}$	$-\frac{\pi}{2}$
y							



Features of the parent tangent function:

passes through:

each "branch" is:

x-intercepts (zeros):

vertical asymptotes:

"guide points":

Properties of Tangent Functions

Suppose $y = a \tan b\theta$, where $b > 0$, and θ in radians.

- The period of the function is _____
- 1 cycle occurs in the interval from _____ to _____
- There are vertical asymptotes at each end of the cycle.
- The pattern is "asymptote, -a, zero, a, asymptote".

Example 2: Graphing a Tangent Function

Sketch 2 cycles of each tangent function.

1. $y = \tan \pi \theta$

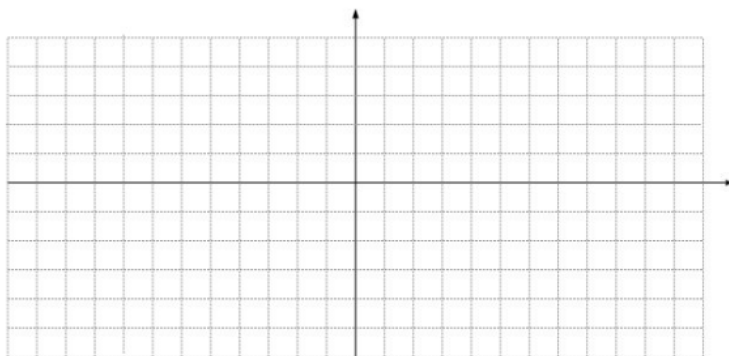
period:

1 cycle: from _____ to _____

VA:

2 guide points:

pattern:



2. $y = \tan 3\theta$

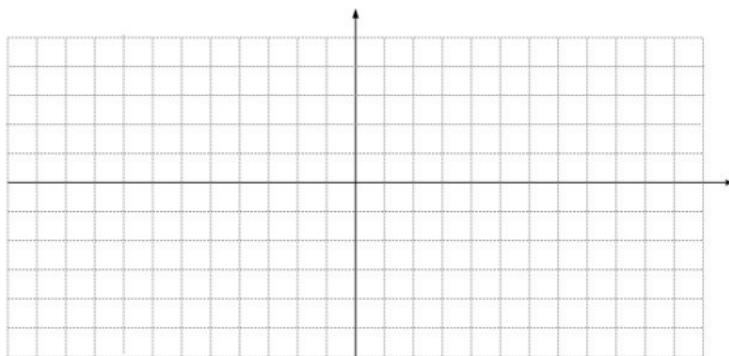
period:

1 cycle: from _____ to _____

VA:

2 guide points:

pattern:



3. $y = \tan\frac{\pi}{2}\theta$

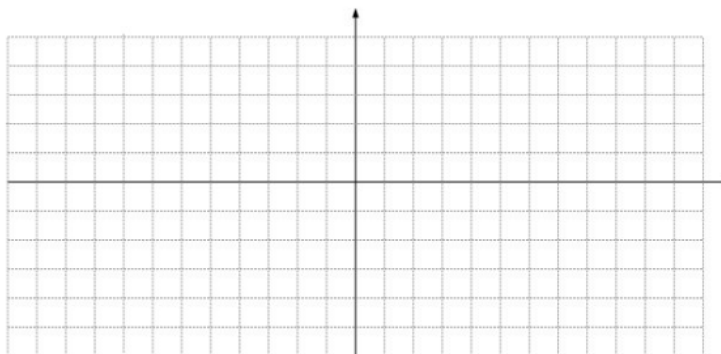
period:

1 cycle: from _____ to _____

VA:

2 guide points:

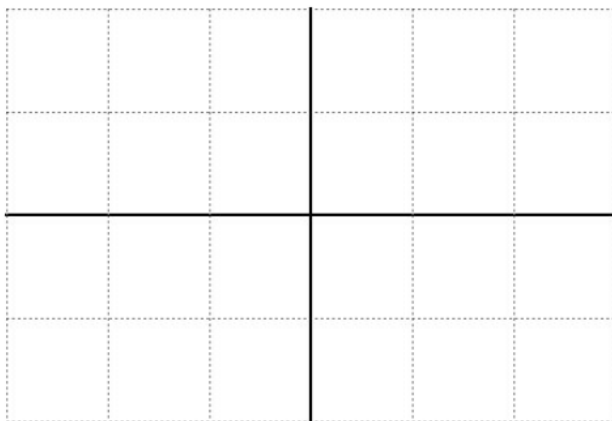
pattern:



13.7 Translating Sine and Cosine Functions

Warm Up:

1. Graph $y = \tan \theta$... again! (Try not to peek at prior notes.)



Features of the tangent function:

passes through:

each "branch" is:

x-intercepts (zeros):

vertical asymptotes:

"guide points":

2. Compare each pair of equations. State the translations (horizontal, vertical) involved.

a. $y = 2x$, $y = 2x + 5$

b. $y = |x|$, $y = |x + 3|$

c. $y = x^2$, $y = x^2 - 4$

d. $y = |x - 2| + 1$

e. $y = x^2$, $y = (x + 3)^2 - 6$

d. $y = f(x)$, $y = f(x - h) + k$

I. Graphing Translations of Trigonometric Functions

Phase shift - the horizontal translation of a function. If $f(x)$ is the "parent", then $f(x-h)$ translates horizontally h units. For example: $f(x-1)$ translates _____, $f(x+3)$ translates _____.

Vertical shift - the vertical translation of a function. If $f(x)$ is the "parent", then $f(x)+k$ translates vertically k units. For example: $f(x) - 1$ translates _____, $f(x) + 3$ translates _____.

Example 1: Identifying Phase Shifts and Vertical Shifts

What is the value of h and k in each translation? Describe the shift i.e. "3 units to the left".

a. $f(x-2)$

b. $y = \cos(x+4)$

c. $f(t-5)$

d. $y = \sin(x+3)$

e. $f(x) - 2$

b. $y = \cos x + 4$

c. $f(t) - 5$

d. $y = \sin x + 3$

Example 2: Graphing Translations

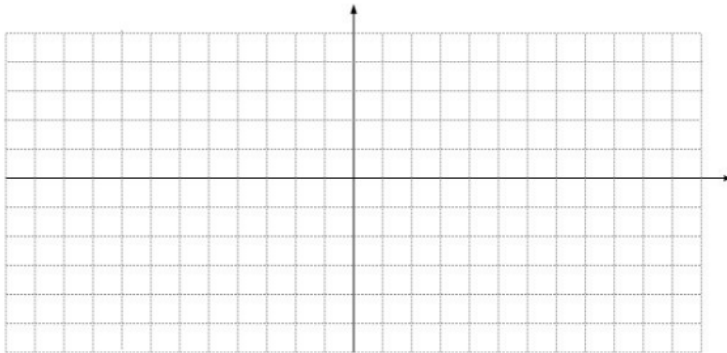
Make a table and then graph the following functions on the same set of axes:

$y = \sin x$

x					
y					

$y = \sin x + 3$

x					
y					

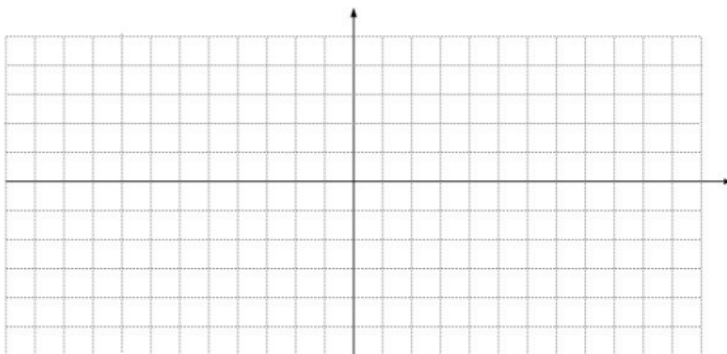


$y = \cos x$

x					
y					

$y = \cos(x - \frac{\pi}{2})$

x					
y					



Example 3: Graphing a Combined Translation

1. $y = \sin(x + \pi) - 2$

amplitude:

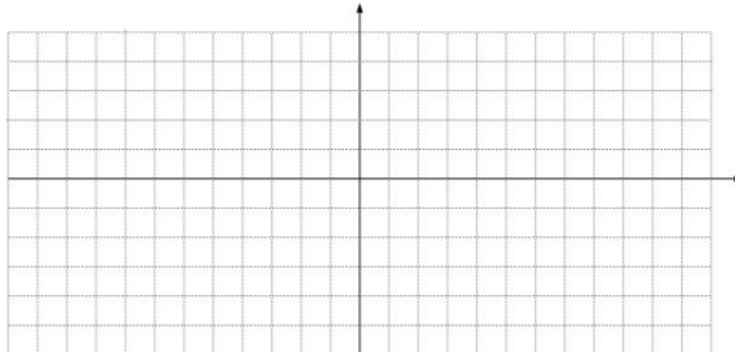
period:

phase shift:

interval spacing on x-axis:

vertical shift:

5 point pattern:



2. $y = 2\cos(x - \frac{\pi}{2}) + 3$

amplitude:

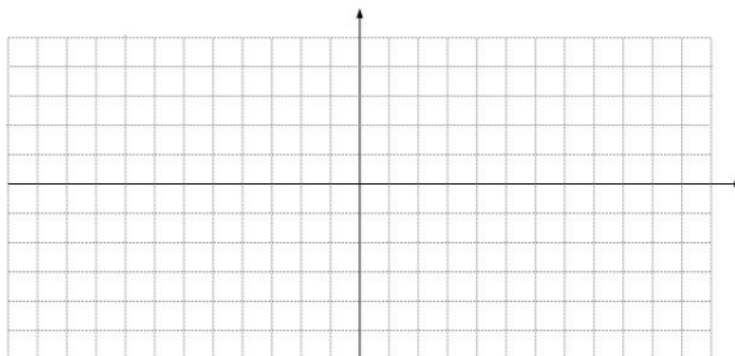
period:

phase shift:

interval spacing on x-axis:

vertical shift:

5 point pattern:



Summary: Families of Sine and Cosine Functions

Parent	Transformed Function
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y=sinx	_____
--------	-------

y=cosx	_____
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amplitude =	h =
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period =	k =
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13.7 Translating Sine and Cosine Functions (Day 2)

Warm Up:

$$y = -3\sin\left(x + \frac{\pi}{2}\right) + 2$$

amplitude:

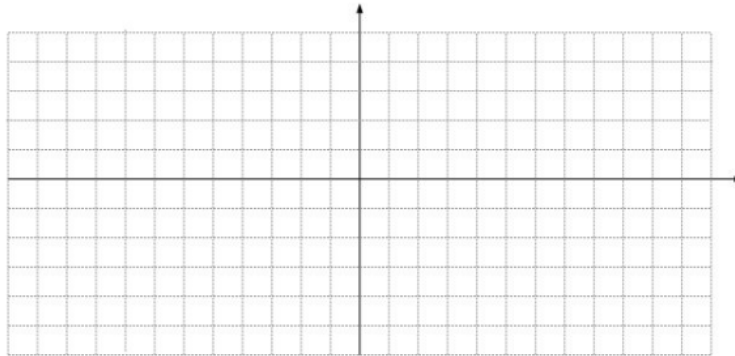
period:

phase shift:

interval spacing on x-axis:

vertical shift:

5 point pattern:



When the phase shift is a pesky number...

$$y = \sin\left(x - \frac{\pi}{3}\right)$$

amplitude:

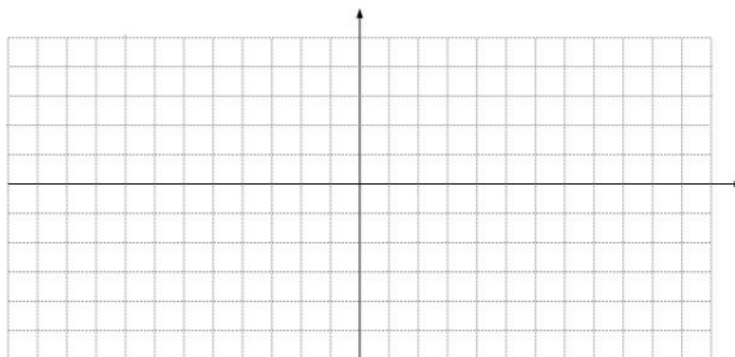
period:

phase shift:

interval spacing on x-axis:

vertical shift:

5 point pattern:



Graphing a translation of $y = \sin 2x$...

$$y = \sin 2\left(x - \frac{\pi}{3}\right) - \frac{3}{2}$$

amplitude:

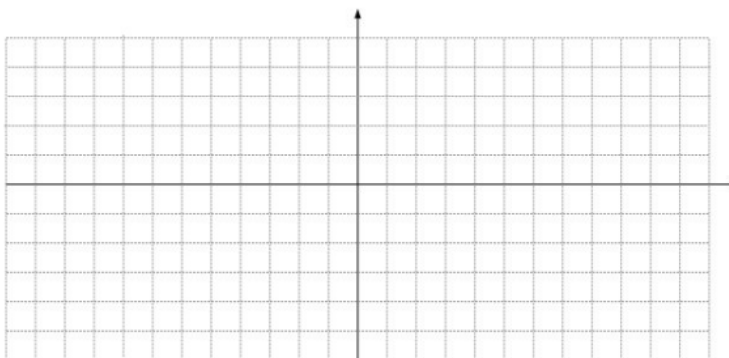
period:

phase shift:

interval spacing on x-axis:

vertical shift:

5 point pattern:



$$y = -3\sin 2\left(x - \frac{\pi}{3}\right) - \frac{3}{2}$$

amplitude:

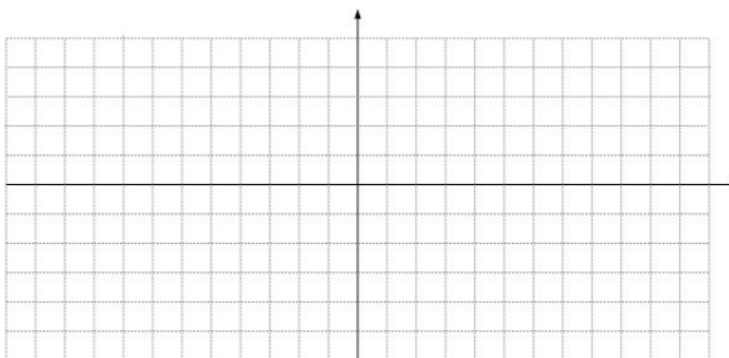
period:

phase shift:

interval spacing on x-axis:

vertical shift:

5 point pattern:



$$y = 2\cos\frac{\pi}{2}(x + 1) - 3$$

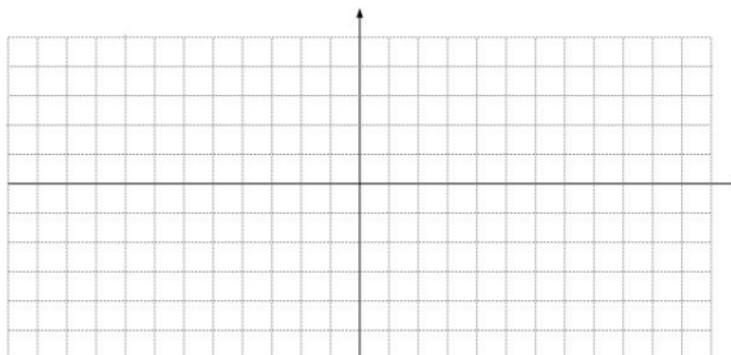
amplitude:

period:

phase shift:

interval spacing on x-axis:

vertical shift:



5 point pattern:

Example 5: Writing a Translation

Write an equation for each translation.

a. $y = \sin x$, π units down

b. $y = -\cos x$, 2 units left

c. $y = \cos x$, $\frac{\pi}{2}$ units up

d. $y = 2\sin x$, $\frac{\pi}{4}$ units right

13.8 Reciprocal Trigonometric Functions**Warm up:**

Find the reciprocal of each fraction:

1. $\frac{9}{13}$

2. $-\frac{5}{8}$

3. $\frac{1}{2\pi}$

4. $\frac{14}{-7}$

5. θ

Name the 3 trigonometric functions you have studied so far:

1. _____

2. _____

3. _____

These 3 trigonometric functions have reciprocals.

Definition: Cosecant, Secant, and Cotangent**Example 1: Using Reciprocals**

a. Use your calculator (degree mode). Round your answer to the nearest hundredth.

$\csc 60^\circ$

$\cot 55^\circ$

$\sec 15^\circ$

b. Suppose $\cos\theta = \frac{5}{13}$. Find $\sec\theta$.c. Suppose $\sin\theta = \frac{-12}{13}$. Find $\csc\theta$.**Example 2: Find The Exact Value**

$\csc 30^\circ$

$\csc 45^\circ$

$\csc 60^\circ$

$\csc 90^\circ$

$\sec 30^\circ$

$\sec 45^\circ$

$\sec 60^\circ$

$\sec 90^\circ$

$\cot 30^\circ$

$\cot 45^\circ$

$\cot 60^\circ$

$\cot 90^\circ$

Example 3: Using Radians

a. Use your calculator (radian mode). Round your answer to the nearest hundredth.

$$\sec(-1)$$

$$\csc(-1.5)$$

$$\sec 2$$

b. Find the exact value.

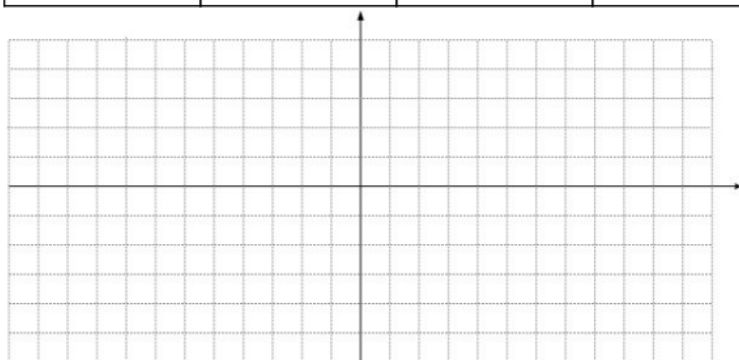
$$\cot \frac{\pi}{3}$$

$$\cot \pi$$

$$\sec 0$$

Example 4: Graph The Reciprocal Trigonometric Functions

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y = sin x					
y = csc x					



x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y = cos x					
y = sec x					

